

## Chapter 2

# The Complexity of Arc Routing Problems

*René van Bevern*  
*Rolf Niedermeier*  
*Manuel Sorge*  
*Mathias Weller*

## 2.1 ■ Introduction

This chapter is devoted to surveying aspects of computational complexity for three central arc routing problems (and their corresponding variants):

- CHINESE POSTMAN, where one asks for a minimum-cost tour traversing all edges of a graph at least once;
- RURAL POSTMAN, which generalizes CHINESE POSTMAN in the sense that *only a subset* of the edges has to be visited; and
- CAPACITATED ARC ROUTING, representing the most general arc routing problem in this chapter, allows more than one vehicle to be used to traverse the edges.

With the exception of the basic version of CHINESE POSTMAN, almost all of these problems are NP-hard; that is, they are computationally intractable problems with respect to the worst-case running times of existing solving algorithms (see Garey and Johnson [41]). Due to their practical importance, however, it is of interest to search for ways to cope with their computational intractability. In theoretical computer science—where strong emphasis is laid on *provable* performance bounds—there are basically two lines of attack:

- polynomial-time approximation algorithms [6, 92, 95], where one trades solution optimality for efficient, polynomial running time instead of exponential, as to be expected for finding exact solutions; and
- parameterized algorithmics and complexity analysis [24, 34, 75], where one seeks to identify small problem-specific parameters that influence the seemingly unavoidable exponential-time behavior. Then, algorithms for finding exact solutions in a running time that is exponential only in the parameter value can be designed.

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